

MODELING UTAH POPULATION DATA

Math 1010 Intermediate Algebra Group Project

According to data from the U.S. Census Bureau, Population Division, the population of Utah appears to have increased linearly over the years from 1980 to 2008. The following table shows the population in 100,000's living in Utah according to year. In this project, you will use the data in the table to find a linear function $f(x)$ that represents the data, reflecting the change in population in Utah.

Estimates of Utah Resident Population, in

	100,000's					
Year	1981 ⁸	1989 ⁴	1993 ⁶	1999 ⁶	2005 ³	2008
x	1	9	13	19	25	28
Population,						
y	15.2	17.1	19	22	25	27.4

Source: U.S. Census Bureau, Population Division

- 5 1. Using the graph paper on the last page, plot the data given in the table as ordered pairs. Label the x and y axes with words to indicate what the variables represent.
- 0 2. Use a straight edge to draw on your graph what appears to be the line that "best fits" the data you plotted. You will only have one line drawn, rather than several pieces of lines
- 4 3. Estimate the coordinates of two points that fall on your best-fitting line. Write these points below.

$$\begin{matrix} (5, 16), & (16, 21) \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

Use the points that you wrote down to find a linear function $f(x)$ for the line. Show your work!

$$m = \frac{21 - 16}{16 - 5} = \frac{5}{11}$$

$$y - 21 = \frac{5}{11}(x - 16)$$

$$y - 21 = \frac{5}{11}x - 7\frac{3}{11} + 21$$

$$y = \frac{5}{11}x + 13\frac{8}{11}$$

$$21 = \frac{5}{11}(16) + b$$

$$21 = 7\frac{3}{11} + b$$

$$-7\frac{3}{11} \quad -7\frac{3}{11}$$

$$13\frac{8}{11} = b$$

$$f(x) = \frac{5}{11}x + \frac{151}{11}$$

4. What is the slope of your line? $m = \frac{5}{11}$

Interpret its meaning. Does it make sense in the context of this situation? Please use complete sentences to respond to these questions.

The rise of the line is five and the run is eleven. This means that the population of Utah increases 500,000 people every 11 years. The number seems to make sense and since the slope is positive, it shows that more people are being born than are dying.

- 3 5. Find the value of $f(45)$ using your function from part 3. Show your work, then write your result in the blank below.

$$\frac{5}{11}(45) + 13\frac{8}{11}$$

$$20\frac{5}{11} + 13\frac{8}{11}$$

$$f(45) = 34\frac{2}{11} \text{ or } \boxed{\frac{376}{11} = y}$$

Write a sentence interpreting the meaning of $f(45)$ in the context of this project.

If x is 45 and represents years, then y must equal population. This means that the population after 45 years would be approximately 3,418,181.

- 3 6. Use your function from part 3 to approximate in what year the residential population of Utah reached 2,000,000. Show your work.

$$20 = \frac{5}{11}x + \frac{151}{11}$$

$$\frac{-151}{11} \quad \frac{-151}{11}$$

$$\frac{69}{11} = \frac{5}{11}x$$

$$\frac{5}{11} \quad \frac{5}{11}$$

$$\boxed{x = 13\frac{4}{5} \text{ or } \frac{69}{5}}$$

Since the starting point for the population estimate table is 1981, one must add $13\frac{4}{5}$ years to get the year 1994, thus in about October ($\frac{4}{5}$'s of the year) the population should reach 2,000,000.

- 4 7. Compare your linear function with that of another student or group.

Comparison function: $f(x) = \frac{3}{8}x + 15$

Is the comparison function the same as the function you wrote down for part 3?

NO

If they are different, explain why.

$\frac{3}{8}$ and $\frac{5}{11}$ are comparable numbers. We got different results because of initial best fit points.

If they are the same, explain why.

- 6 8. In actuality, using a linear growth model for population is not common. Most models are exponential models, due to the fact that most populations experience relative growth, i.e. 2% growth per year. Linear models for nonlinear relationships like population work only within a small time frame valid close to the time of the data modeled. Discuss some of the false conclusions you might reach if you use your linear model for times far from 1980-2008. With a linear model, the slope will always be the same because m never changes. With an exponential model, the slope is always increasing from year to year. The line on which points are graphed for a linear model will always be going up straight, assuming that the population is increasing. In an exponential model, when a pop. starts at say 100 and increases 2%. the next year, it goes up to 102 people. Once the pop. becomes so great, the line will curve drastically because, for example, a 2% increase of 100,000 is 2,000 people. Thus, an exponential model is a more precise graphing method when it comes to graphing. (10)

